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## **SPATIAL ABILITIES: A GROUP OF BASIC WORKPLACE SKILLS DEVELOPED THROUGH GEOGEBRA 3D**

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### **Another PISA shock from Hungary: failure in mathematics partly explained by deficient spatial skills**

Results of the 2012 survey of the Program for International Student Assessment (PISA, 2013) revealed a sharp decline in performance both in Mathematics and in Science for Hungary. The first analyses of the causes of yet another decrease of performance in these, economically important areas have revealed low educational expenditure, too frequent curricular reforms and outdated teaching methodology. A detailed analysis of tasks where performance is especially low were mostly those involving spatial abilities. Hungarian art education has traditionally focused on the acquisition of classic European rules of representation (rendering methods of linear perspective) and included knowledge previously taught by Descriptive Geometry, once an individual discipline, later a part of Mathematics and Art and Visual Culture (the name of the discipline for art education in Hungary). In the last decade, Art became an elective from age 13 (Grade 7), and thus the cohort taking the PISA test is the first that may have reached secondary school without any art lessons in the period decisive for the development of spatial abilities: 13-16 years.

Task types “Shape and Space” in the PISA assessment focused on one or more basic skill components (reconstruction, mental rotation, change of viewpoints, estimation of surface size, etc.) required the utilisation of spatial abilities in everyday situations. These tasks were typically included in the Art curriculum (cf. an overview in Kárpáti, Babály and Budai, 2013), therefore, increase in performance cannot be expected solely from a more efficient education in Mathematics. Moreover, spatial skills are among those core abilities required for the majority of skilled jobs and also a wide variety of professions. Therefore, the demonstrated lack of these skills may emphasize the importance of educational efforts aimed at the development and assessment of the perception of space and the creation and modelling in space. In a recent national assessment project, we developed a visual skills framework for curriculum innovation (Kárpáti & Gaul, 2011).

Spatial skills identified and evaluated in this Framework are as follows: Spatial perception, Orientation in space, Experiencing space, identifying spatial qualities, Interpretation of spatial structures, longitudinal and cross sections, Spatial representation (2D), Representing spatial

qualities based on visual perception, Representing positions in space (2D), Creation of spatial sensations caused by the organisation of visual elements (e.g. rhythm, balance), Representation of changing experiences of space through time, Reconstruction of space, Reproduction of space, abstraction, Creation of spatial objects (2D and 3D), Design, Modelling, Creation, Construction. Information and Communication Technologies may be instrumental in the enhancement of this competence area. This paper presents a research project aimed at developmental, authentic assessment of spatial abilities through the new, 3D version of the GeoGebra software.

## **Dynamic visualisation through 3D GeoGebra**

GeoGebra, this innovative, dynamic visualization software was created by Markus Hohenwarter and originally intended for use in secondary level science and mathematics education. It is available as an open source application and can be installed on any platform that is suitable to run Java. Today, however, it is available in 62 languages in 122 institutions of 190 countries. With more than 45,000 online study material available, about 5.5 million copies of the open source software were being used in schools in 2012. Thousands of volunteer developers broaden the range of applications daily.

Its latest version, GeoGebra 5.0 ([http://wiki.geogebra.org/en/Release\\_Notes\\_GeoGebra\\_5.0](http://wiki.geogebra.org/en/Release_Notes_GeoGebra_5.0)) includes 3D functionalities and is ideally suitable for digital creation in space. Perhaps the most important feature of this version is that it connects different representations of objects with their geometric display and algebraic description. GeoGebra is a dynamic system because users get a virtual designing kit with the program that enables them to visualize any spatial problem. Unlike designing on paper, the initial objects (points, straight lines...) can be freely moved while the objects dependent upon them move along with them based on their geometrical connections. Thus, students practicing mental rotation can actually rotate a linear representation of a cube and see its shape changing according to the change of perspective. Discovery learning at its best, the system can also be used for testing the level of spatial perception.

Through an integration of the GeoGebra 5.0 Beta and 4.x, we may develop virtual learning and evaluation task sheets that activate spatial skills in an authentic setting resembling manipulation and orientation in real space. The task shown on Figure 1 requires the selection of a cross section from among alternatives. After having made a selection, the student receives immediate, dynamic feedback about the appropriate solution and may understand if and why his or her choice was wrong.

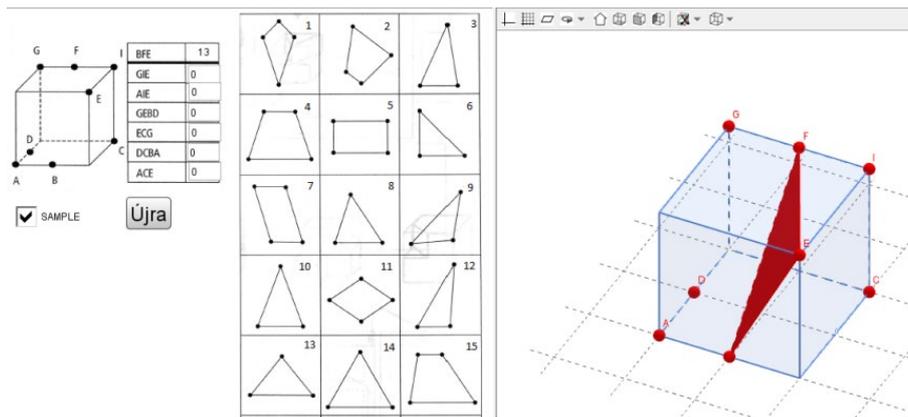


Figure 1. Let us select the suitable shape, which the cube is on his edges, we get some of the binding of granted dots

On Figure 2, we show a dynamic task sheet that develops high level abstraction and reconstruction of space. The student is asked to create a two-dimensional figure, and rotate it around a spatial axe. Several tasks can be built on this basis. The parallel sides of the string trapezium 4 cm and 2 cm, his stems 3 cm long. The shape rotates around the axis of the symmetry. How large are his surface and his volume for the object got so? (It is a truncated coin!)

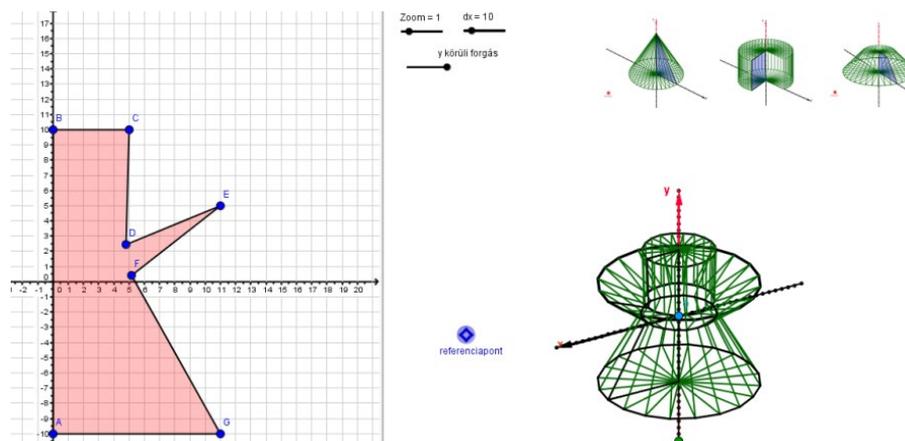
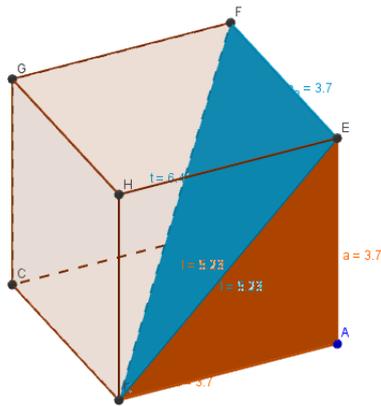


Figure 2. Creation of a spatial figure through rotation of a two-dimensional plane

The highly interactive task represented on Figure 3 may be utilised to develop skills of solving tasks of spatial geometry.



él hossza:

Tekintsük például a DAE derékszögű háromszöget!  DAEΔ

Erre a háromszögre felírható Pitagorasz-tétele.

A két befogót a-val, az átfogót l-el jelölve: (l a kocka egyik lapátlója)

$$a^2 + a^2 = l^2$$

Behelyettesítve a Pitagorasz-tételbe:  $3.7^2 + 3.7^2 = l^2$

$$13.69 + 13.69 = l^2$$

$$27.38 = l^2$$

Vonjunk mindkét oldalból négyzetgyököt, így:

$$5.23 = l$$

A 3.7 cm élhosszúságú kocka lapátlójának hossza tehát 5.23 cm.

A testtíró meghatározásához szükségünk van az utóbb kiszámolt lapátló hosszára is. Tekintsünk most egy olyan derékszögű háromszöget, amiben a lapátló, a testtíró és a kocka éle is:

Tekintsük a DEF derékszögű háromszöget!  DEFΔ

A DEF derékszögű háromszög derékszöge az E csúcsnál van. Pitagorasz-tételt ismét alkalmazhatjuk, mert ismert az a oldal

l helyére

Figure 3. Interactive task sheet for calculation

Here, students may determine the length of the edge length of the cube and may follow the steps of the computation of the size of the diagonal. Help is provided as it is needed, in a step by step manner, so students may use the task as a support for solving a home assignment or practicing computation techniques. The 3D, dynamic representation of the figure helps the development of mental manipulation as it enables constant comparisons between a cross section and a shape in space. As all the parameters of the task can be varied, it is easy for the teacher to develop another task based on another geometrical shape, like a pyramid or sphere.

As the cube is the figure most frequently used in Mathematics education, the tasks represented on Figures 4 and 5 can be optimally used to develop mental manipulations necessary for the reconstruction of a figure based on its floor plan, or the selection of the correct floor plan related to a figure. Here, dynamic potentials of GeoGebra are optimally used to support mental imagery.

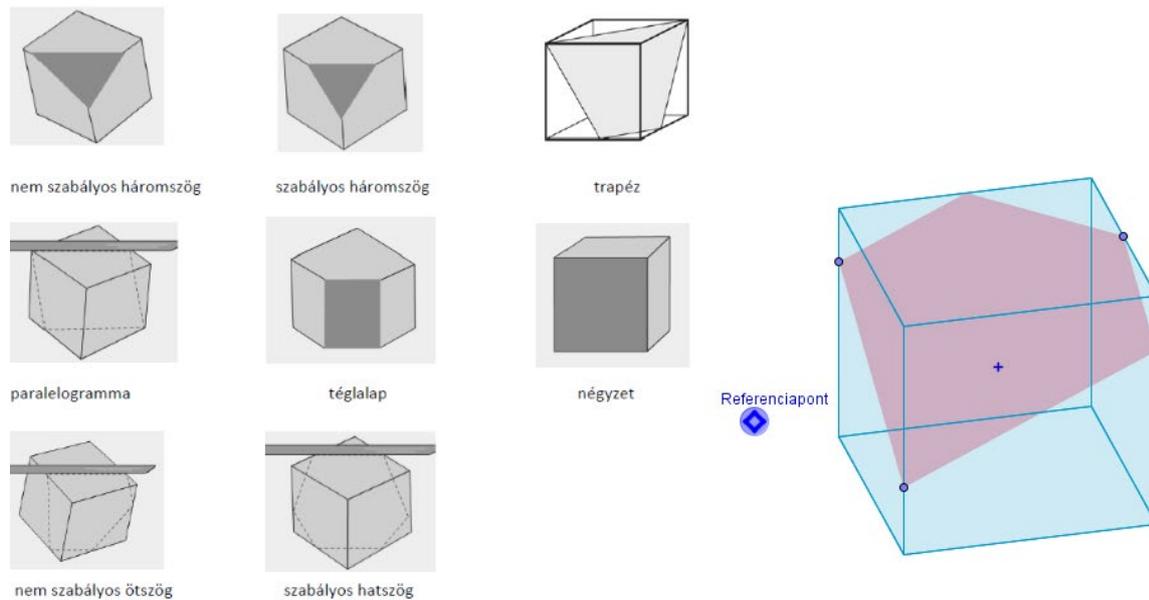


Figure 4. Intersection of a cube

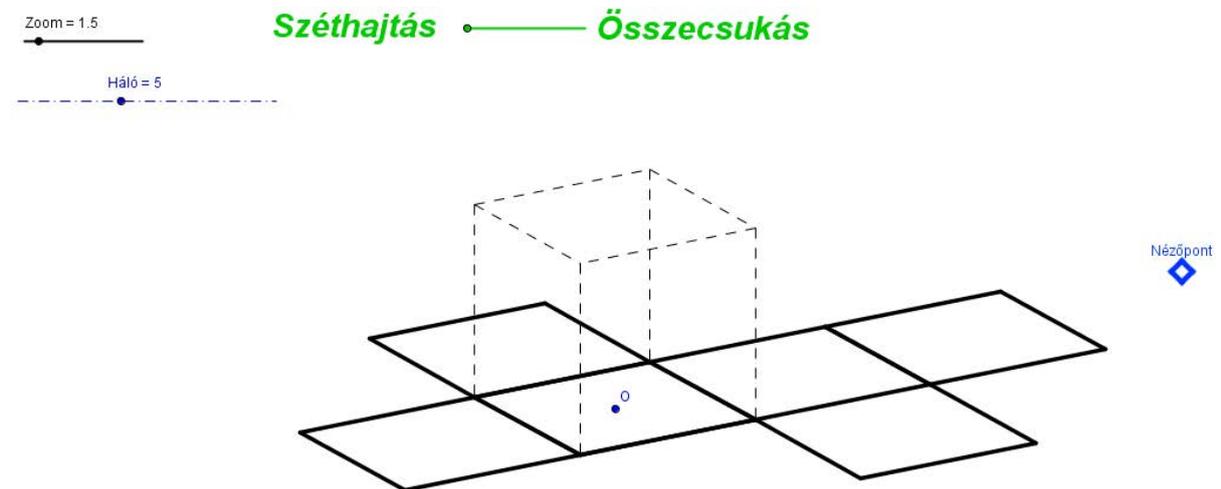


Figure 5. All nets of a cube

It is important to go beyond orthogonal axonometric tasks and show examples that involve central axonometric knowledge as well. These tasks pave the way to the perception and understanding of spherical geometry and the 4d “hypercube” (It is an object, that every “sides” is a 3D cube). These tasks do not only enhance spatial skills but also build a motivation to acquire them on higher levels.

3D GeoGebra is ideally suited to represent buildings and their floor plans from different axes and thus give an idea about a historic building in space (Figure 6):

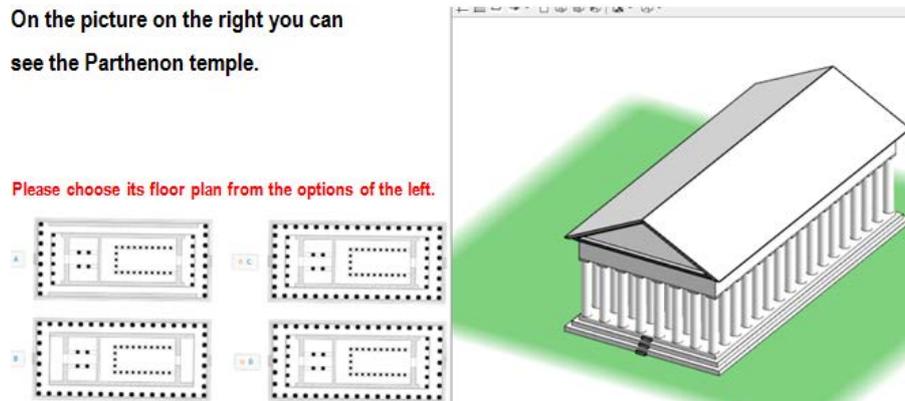


Figure 6. Floor plan and reconstruction of the Parthenon  
The image of the building may be rotated to experience views from various angles

## Assessment of spatial skills

Manipulating objects in space through two-dimensional abstractions has been accepted as a valid means of identifying spatial skills and assessing them – however, working with generations of students deeply immersed in multimedia technology, we found this solution unauthentic and idiosyncratic. Edutainment and gaming applications (like those developed by the Quest to Learn project) have long been using sophisticated virtual spaces that activate skills ranging from orientation to memory, manipulation to construction. KINECT transmits real movement to virtual space and thus provides authentic orientation experiences. The Leonar3Do software enables users to manipulate in real space and create 3D images that can be shown through a 3D printer as sculptures or objects. Manipulation in virtual space is being employed at the Harvard Mental Imagery Lab where a spatial aptitude test is developed using Virtual Reality and Augmented Reality solutions. Our methodological objective is to integrate these digital solutions in educational assessment in the visual arts.

We employ digital technology in two forms: first, to provide students with a personalised, flexible, online practicing and testing environment. Second, we started experimenting with three-dimensional (3D) software solutions that provide authentic methods for creation, manipulation and perception of space in a dynamic virtual environment. In this paper, we give a brief account of our first results comparing traditional and innovative evaluation methodologies.

In order to contextualise visual skills as important and assessable components of education, we joined the “Development of The Assessment of Cognitive and Affective Skills and Abilities Project” of Szeged University. In the first phase of the Visual Literacy sub-project, we developed and piloted a set of paper-based and digital tasks. Later, the best tasks were included in eDIA, the *online, adaptive testing environment* of the project that provides an easy-to-use, freely available for all Hungarian schools testing environment. The spectacular visual appearance of the tasks of eDIA makes it an enjoyable visualisation tool that makes it

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easier to comprehend spatial problems than black-and-white, abstract axonometric projections in traditional paper-based tests. During electronic assessments, students work in an environment that resembles social web sites as well as gaming applications. Usage studies show that they can orientate in the menu without effort. In the “Visual Culture” task package, we always provide practice items that show manipulation options and also a voiceover for slow readers. Digital images provide a life-like representation of space and reproduce complex spatial situations accurately.

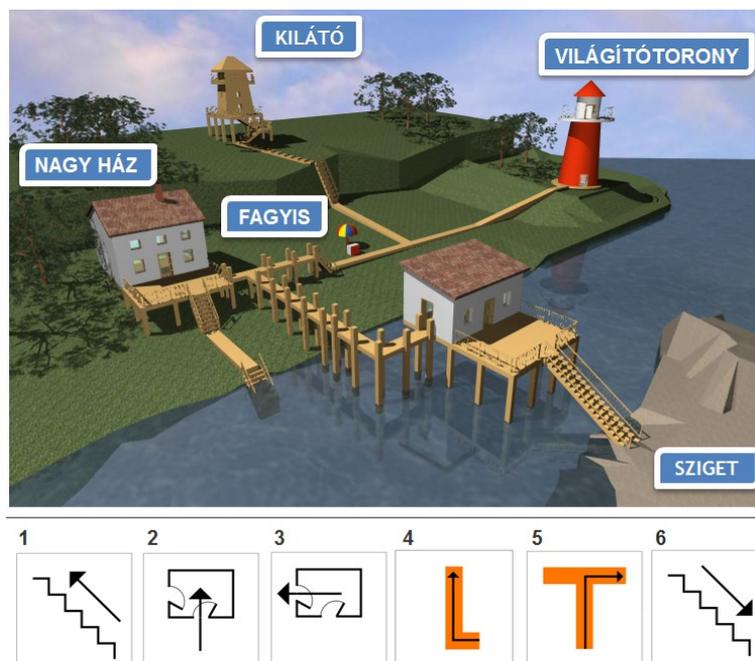


Figure 7. Finding your way around in a virtual space, using directions represented by signs (shown below). Task for 4th graders (age: 10 years)

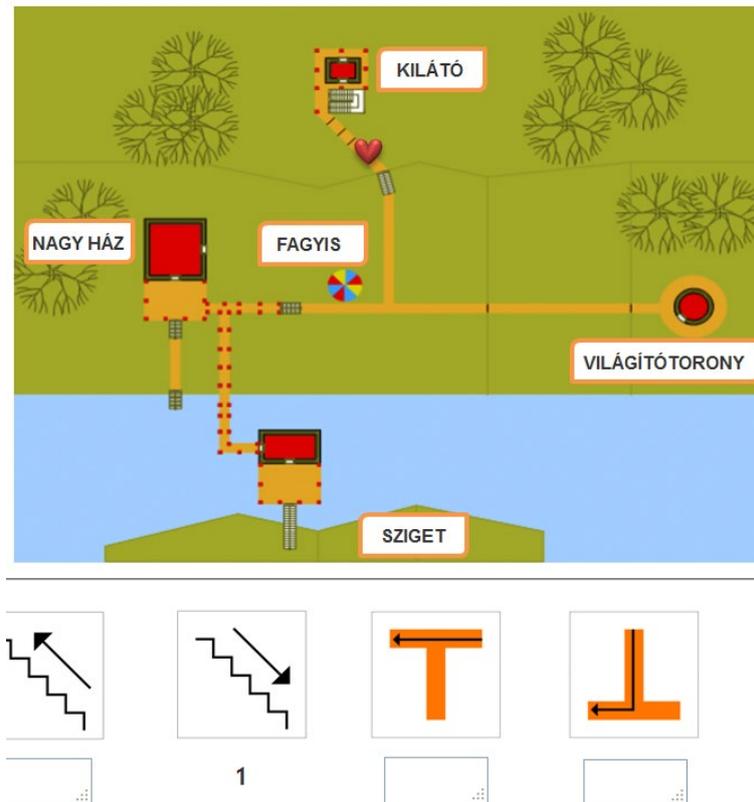


Figure 8. A more difficult version of the spatial orientation task in Figure 1 (for 6th graders, aged 12): the virtual space is depicted as a map, directions represented by signs.

Task response types include marking, colouring and moving images, entering text, joining text and picture or forming groups of items. Cognitive skills involved in perception, design and creation are targeted simultaneously, just like in real life. Visual skills are in the focus, but other competences are also targeted, revealing the *interdisciplinary significance of art education*.

In eDIA, results of Art and Visual Culture may be compared with four core disciplines (Mathematics, Mother Tongue, Science and Foreign Languages) as well as eleven other areas of studies (including Music and Media Arts) to reveal correlations and cognitive, affective and psychomotor gains resulting from education through art. In its final form, the eDIA-system will monitor personal development, offer tasks for individual skill enhancement based on previous results. Art teachers may thus design individualised teaching-learning processes that supports talent development and caters for special needs (like mental or psychomotor deficits) at the same time.

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